CONSTELLATION-X AND COSMOLOGY WITH NUMBER COUNTS OF GALAXY CLUSTERS

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Cosmological studies based on the number counts of galaxy clusters are — or soon will be — limited by systematics, not statistics. The dominant source of uncertainty is a bias and scatter in the determination of total mass. A major contribution of *Constellation*-X to this field is twofold. First, low-scatter $M_{\rm tot}$ proxies can be provided by snapshot observations of complete samples of high-redshift clusters. Second, biases in the $M_{\rm tot}$ determination can be determined by detailed observations of representative cluster subsamples.

1 Mass accuracy requirements

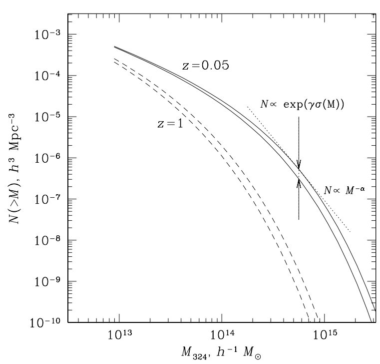

FIG. 1.— The mass function for two redshifts, $z = 0.05$ (solid) and $z = 1$ (dashed), and for $\sigma_8 = 0.8$ and 0.75 (top and bottom, respectively) shows strong evolution with redshift, steep dependence on mass, and exponential sensitivity to the amplitude of matter density perturbations at a given mass scale.

Using counts of galaxy clusters as a tool for cosmological studies is based on the exceptional sensitivity of the cluster number density to the amplitude of the linear density perturbations. Evolution of the cluster number density at high redshifts constrains the growth of density perturbations which, together with the distance-redshift relation, is a prime dark energy observable [1]. Ideally, a large cluster survey can provide very tight constraints on the dark energy equation of state [2] but the requirements for the accuracy of the total mass measurements are stringent.

Theoretical models for the mass function of dark matter halos in all variants of CDM cosmology are well-developed and accurately calibrated by numerical simulations [3, 4, 5, 6]. The models show that the comoving number density of clusters is a steep function of mass and is exponentially sensitive to the amplitude of matter density perturbations at the given mass scale (Fig. 1). The mass function is usefully characterized by two numbers, the local power law slope, α, and the sensitivity of the cluster number density, γ, to the amplitude of linear density perturbations, $\sigma(M)$, on scale M:

$$N(>M) \propto M^{-\alpha}, \quad \gamma = \frac{d\ln N(>M)}{d\sigma(M)}, \tag{1}$$

The values of these quantities computed for a typical combination of the cosmological parameters are given in Table 1.

TABLE 1 MASS FUNCTION PARAMETERS FOR DEFAULT ΛCDM COSMOLOGY

	$z = 0.05$			$z = 1$		
M, $h^{-1}\,M_\odot$	α	γ	σ	α	γ	σ
1×10^{14}	1.70	3.6	0.87	2.95	15.8	0.55
2×10^{14}	2.04	5.8	0.76	3.71	23.5	0.48
4×10^{14}	2.65	10.0	0.64	4.89	37.4	0.40
6×10^{14}	3.13	13.8	0.58	5.80	49.2	0.36

1.1 Requirements for systematic biases

Using α and γ we write simple expressions for the uncertainties in the amplitude of perturbations measured from a cluster sample of size N,

$$\Delta\sigma(M) = \left[\left(\frac{1}{\gamma}\,\frac{1}{\sqrt{N}}\right)^2 + \left(\frac{\alpha}{\gamma}\,\frac{\Delta M}{M}\right)^2\right]^{1/2}, \tag{2}$$

where the two terms represent the Poisson noise and contribution from a systematic mass measurement bias. Statistical errors dominate if

$$\Delta M/M < (\alpha\,\sqrt{N})^{-1}. \tag{3}$$

For example, for $N = 1000$ and $\alpha = 2.65$ (corresponding to a mass threshold of $4\times10^{14}\,h^{-1}\,M_\odot$ at $z = 0$), $\Delta M/M < 1.2\%$. Similarly, for $N = 50$ clusters and $\alpha = 4.9$ (typical parameters for $z = 1$), $\Delta M/M$ should be smaller than 3%. Note that for growth factor measurements, the requirements for $\Delta M/M$ are mostly for *relative* calibration of the high- and low-z measurements (i.e., both can have small bias as long as the bias is redshift-independent).

If these requirements are satisfied and thus uncertainties are dominated by Poisson statistics, not systematics, $\sigma(M)$ can be measured quite accurately, $\Delta\sigma \sim (\gamma\,\sqrt{N})^{-1}$, which gives $\Delta\sigma = 0.0032$ with 1000 clusters at $z = 0$ (assuming $\gamma \approx 10$ which corresponds to a mass threshold of $4\times10^{14}\,h^{-1}\,M_\odot$, Table 1).

1.2 Requirements for individual mass measurements

Even in the absence of systematic biases, measurement scatter in mass estimates for individual objects can affect the derived cluster number densities [7]. The effect of the scatter is to smooth the mass function model with a kernel. For a power law mass function and for a log-normal distribution of the mass estimates with constant scatter, $\delta_{\ln M}$, the power law slope is not affected, but its normalization is increased by a factor

$$\text{bias due to scatter} = \exp(\alpha^2\,\delta^2_{\ln M}/2) \tag{4}$$

If the scatter is known a priori, the resulting bias can be corrected. However, if $\delta_{\ln M} > \sqrt{2}/\alpha$, the effect can become uncontrollably large. The requirement for individual mass measurements is then

$$\delta M/M \lesssim 1/\alpha, \tag{5}$$

which corresponds to $\approx 20\%$ accuracy for a mass threshold of $4\times10^{14}\,h^{-1}\,M_\odot$ at $z = 1$.

Random scatter in mass can be both statistical and intrinsic, the latter representing, e.g., scatter in a mass-observable relation. The statistical component is easy to estimate. Intrinsic scatter is a more serious problem if not known a priori (which is usually the case). Its internal estimates are limited by the sample size, $\Delta\delta^2 = \delta^2\,(2/N)^{1/2}$, and so the resulting number density uncertainty is smaller than the Poisson error if $1/\sqrt{N} > \Delta\delta^2\,\alpha^2/2 = \delta^2\,\alpha^2\,(2/N)^{1/2}/2$ (we expand (4) assuming $\alpha^2\delta^2_{\ln M}/2 \ll 1$), or

$$\delta M/M < 2^{1/4}/\alpha, \tag{6}$$

i.e. approximately the same requirement as eq.(5).

To summarize, the typical tolerance for mass measurements is 20% – 30% for individual objects, and a few percent for redshift-dependent systematic biases. If these requirements are met, the statistical accuracy of samples of $\sim$ 100 clusters in several redshift intervals provides interesting constraints on the dark energy equation of state. The required accuracies can be achieved via the survey "self-calibration" or by obtaining high-quality data for each object.

2 Self-calibration

Large-area cluster surveys provides several statistically independent but theoretically degenerate sources of information:

- the distribution of the number of detected clusters as a function of redshift, dN/dz;
- the distribution as a function of mass at each redshift, dN/dM;
- the correlation function of the spatial distribution of detected clusters. $\xi(r)$.

Large surveys are so powerful statistically that the cosmological parameters can be constrained with, e.g., only dN/dz and the rest of information is used to calibrate the cluster mass scale [2, 7]. This process is referred to as *self-calibration*.

The ability for self-calibration quickly degrades if there is a large scatter between the cluster mass and raw observable [7], which is the case, e.g., in the $M_{\rm tot} - L_x$ relation or in the relation between $M_{\rm tot}$ and the peak SZ signal [8].

One obvious solution is to use a sample selection based on a low-scatter proxy for $M_{\rm tot}$, such as the cluster temperature. Large effective area of *Constellation-X* makes it possible to measure T_x to $\sim$ 10% accuracy with $\sim$ 1 ksec exposures. Therefore, it is feasible to reobserve *all* distant clusters detected in surveys such as SPT, or Plank, or a 10,000–20,000 deg^2 X-ray survey of moderate sensitivity.

3 Mass calibration via weak lensing

Weak lensing is an attractive method to determine $M_{\rm tot}$ in galaxy clusters. There are limitations to this method related to both the observing capabilities (seeing, field of view, foreground bright objects), and more fundamental problems such as a small number of background galaxies behind distant clusters and more seriously, projection of large-scale filamentary structure ([9] and later work).

These problems limit the accuracy of the weak lensing measurements for individual objects. However, they should average out when the results for a large number of objects are combined. Therefore, the absolute mass scale can, in principle, be calibrated by stacking analysis of the weak lensing sheer for clusters with a given value of an $M_{\rm tot}$ proxy. Just as for self-calibration, using low-scatter proxies provided by *Constellation-X* observations would be a great advantage.

4 High-quality $M_{\rm tot}$ proxies

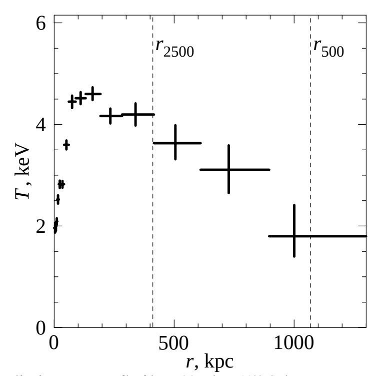

FIG. 2.— *Chandra* temperature profile of the $z = 0.057$ cluster A133. Such temperature measurements are sufficient for hydrostatic $M_{\rm tot}$ estimate within r_{500} with a 9% uncertainty. This kind of data quality will be achievable with *Con-X* for high-redshift clusters.

High-quality X-ray observations provide multiple observables for galaxy clusters. In addition to the total luminosity, L_x, and average temperature, T_x, the spatially resolved spectral data provides the profiles of density and temperature of the intracluster gas. For dynamically relaxed clusters, $\rho_{\rm gas}(r)$ and $T(r)$ give $M_{\rm tot}$ via the hydrostatic equilibrium equation. For example, the quality of the $T(r)$ measurement shown in Fig.2 is sufficient to determine $M_{\rm tot}$ with a 9% uncertainty near r_{500}.

The hydrostatic mass estimates cannot be applied for non-relaxed clusters. However, $\rho_{\rm gas}(r)$ and $T(r)$ still can be combined into high-quality $M_{\rm tot}$ proxies. One example, Y_X, is designed to approximate the integrated Sunyaev-Zeldovich signal from the cluster. SZ signal is proportional to the so called Y-parameter,

$$Y_{SZ} \propto \int \rho_{\rm gas}\,T\,dV, \tag{7}$$

that represents the total thermal energy of the ICM and is expected to have a tight correlation with $M_{\rm tot}$ if integrated within a sufficiently large radius [8]. Numerical experiments show that Y-parameter is an $M_{\rm tot}$ proxy because it is relatively unaffected by non-gravitational energy sources and sinks (ICM cooling, galaxy formation, SN explosions) and the $Y_{SZ} - M_{\rm tot}$ relation stays close to the self-similar prediction [10],

$$M_{\rm tot} \propto E(z)^{-2/5}\,Y_{SZ}^{3/5}, \tag{8}$$

even when non-gravitational process are in the full swing and the cosmological background is non-self-similar (e.g., between $z = 0.5$ and $z = 0$ in the standard ΛCDM).

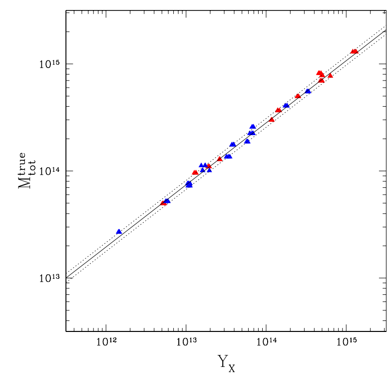

FIG. 3.— Relation between the Y_X parameter and $M_{\rm tot}$ for volume-limited cluster samples from numerical simulations. Red and blue symbols are for simulation outputs at $z = 0$ and $z = 0.6$, respectively. Relation was "de-evolved" using the expected self-similar evolution (eq.8). Dotted lines show 10% deviations about the mean relation. (*From Kravtsov et al., in prep.*

The Y-parameter is "observed" directly through the thermal SZ effect in the radio but not in the X-rays. However, the spatially resolved X-ray measurements can be combined into the X-ray equivalent of eq. 7,

$$Y_X \equiv M_{\rm gas}\,T_{\rm spec}, \tag{9}$$

where $M_{\rm gas}$ is derived from the X-ray surface brightness assuming spherical symmetry, and $T_{\rm spec}$ is from the single-temperature fit to the integrated X-ray spectrum. Numerical simulations show (Fig.3)that Y_X performs similarly or better than Y_{SZ} as an $M_{\rm tot}$ proxy — there is a low (< 10%) scatter in the scaling relation, even for very non-relaxed clusters, and the evolution with z follows the self-similar prediction (eq. 8). Detailed discussion of Y_X as an $M_{\rm tot}$ proxy will be presented in Kravtsov et al. (in preparation).

Using high-quality $M_{\rm tot}$ proxies, i.e. those that are

- well-justified theoretically;
- stable with respect to details of the ICM physics;
- straightforwardly derived from the data;
- reliably computed in the simulations;

alleviates the need for self-calibration in a cluster survey and thus provides an alternative strategy for realization of the cosmological tests based on the growth of structure.

5 Calibration of numerical simulations

Galaxy clusters are unique astronomical objects in the sense that they can be "computed" *ab initio* in the computer models. Current codes are highly accurate and sophisticated, and their output clusters often looks remarkably realistic. Numerical simulations can be used to study how accurately cluster properties are recovered from the data; to estimate selection biases; to provide full theoretical foundation for mass estimators such as Y_{SZ} or Y_X, and so on. Even the ability of cluster surveys to self-calibrate is, ultimately, to be demonstrated using large-volume numerical experiments.

If numerical computations are employed in the cosmological work, the simulated clusters should be sufficiently realistic. *Constellation-X* can provide cosmology-independent data such as the profiles of gas temperature and metallicity, and the shape of $\rho_{\rm gas}(r)$, which can be used to verify that the simulations indeed produce realistic clusters at all redshifts.

6 Conclusions

The distance-redshift relation and the growth of density perturbations are two prime dark energy observables. Clusters are highly effective in constraining the growth of structure because their number density as a function of mass is strongly (exponentially) sensitive to the amplitude of density perturbations. However, the strong sensitivity of the cluster number density to the underlying cosmology also implies that the test is sensitive to systematic errors in the mass determination. Thus, the full statistical power of large cluster samples can be exploited only if any systematic biases in the mass scale are less than a few percent.

Constellation-X will not be able to find large numbers of distant clusters due to its narrow field of view. However, *Con-X* can contribute enormously by providing high-quality data for distant clusters detected in the wide-area X-ray or SZ surveys.

References

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